

Approximate the sum of the series to the indicated accuracy

① $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$ (error < 0.01) ② $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}$ (error < 0.001)

③ Is the 50th partial sum S_{50} of the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ an over estimate or an underestimate of the total sum? Explain

④ Find the radius of convergence & interval of convergence

(a) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ (b) $\sum_{n=0}^{\infty} \frac{n}{4^n} (2x-1)^n$ (c) $\sum_{n=2}^{\infty} (-1)^n \frac{(2x+3)^n}{n \ln n}$

(d) $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-1}}{(2n-1)!}$

⑤ Find the power series representation for each of the following functions and determine its radius of convergence

(a) $f(x) = \frac{1}{(1+x)^3}$, (b) $f(x) = x \ln(1+x)$, (c) $f(x) = \tan^{-1}(2x)$

⑥ Evaluate the following indefinite integrals as power series

$\int \frac{x}{1+x^3} dx$ $\int \tan^{-1}(x^2) dx$

(7) Evaluate $\int_0^{1/2} \frac{1}{1+x^7} dx$ using power series representation

How many terms do we need to achieve an accuracy of 10^{-10} or better

(8) Obtain the Maclaurin series for

(a) $x^2 \cos(x)$ (b) $x \sin(x/2)$ (c) 2^x

(9) Find the sum of the series

(a) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n!}$ (b) $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n} (2n)!}$ (c) $\sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)!}$

(10) What is the maximum error possible in using the approximation $\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!}$ when $-0.3 \leq x \leq 0.3$?