

Maclaurin and Taylor Series Representations

$$1) e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}, -\infty < x < \infty$$

$$2) \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}, -\infty < x < \infty$$

$$3) \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}, -\infty < x < \infty$$

$$4) \frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots = \sum_{n=0}^{\infty} x^n, -1 < x < 1$$

$$5) \frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \dots = \sum_{n=1}^{\infty} nx^{n-1}, -1 < x < 1$$

$$6) \frac{1}{1+x} = 1 - x + x^2 - x^3 + \dots = \sum_{n=0}^{\infty} (-x)^n = \sum_{n=0}^{\infty} (-1)^n x^n, -1 < x < 1$$

$$7) \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}, -1 < x \leq 1$$

$$8) \ln x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \dots = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-1)^n}{n}, 0 < x \leq 2$$

$$9) \tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}, -1 \leq x \leq 1$$

$$10) (1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} x^3 + \dots = 1 + \sum_{n=1}^{\infty} \frac{\alpha(\alpha-1)(\alpha-2)\dots(\alpha-n+1)}{n!} x^n,$$

[$-1 < x < 1$ for $\alpha \neq$ a nonnegative integer and $-\infty < x < \infty$ for $\alpha =$ a nonnegative integer]

For example:

$$\bullet (1+x)^3 = 1 + 3x + \frac{(3)(2)}{2!} x^2 + \frac{(3)(2)(1)}{3!} x^3 = 1 + 3x + 3x^2 + x^3$$

$$\bullet (1+x)^{1/2} = 1 + \frac{1}{2}x + \frac{(1/2)(-1/2)}{2!} x^2 + \frac{(1/2)(-1/2)(-3/2)}{3!} x^3 + \dots = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \dots$$

$$\bullet (1+x)^{-(1/2)} = 1 + \left(-\frac{1}{2}\right)x + \frac{(-1/2)(-3/2)}{2!} x^2 + \frac{(-1/2)(-3/2)(-5/2)}{3!} x^3 + \dots = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \dots$$